Behavior Indicator of Motion

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1. Introduction

This work deals with the fundamental questions of moving system identification. For this purpose, signals or descriptive functions, for example, are considered with the inputs and outputs of the system.

1.) is there movement?
2.) is there chaotic behaviour?
3.) is there periodic/stationary behaviour?
4.) is there compensatory behaviour?

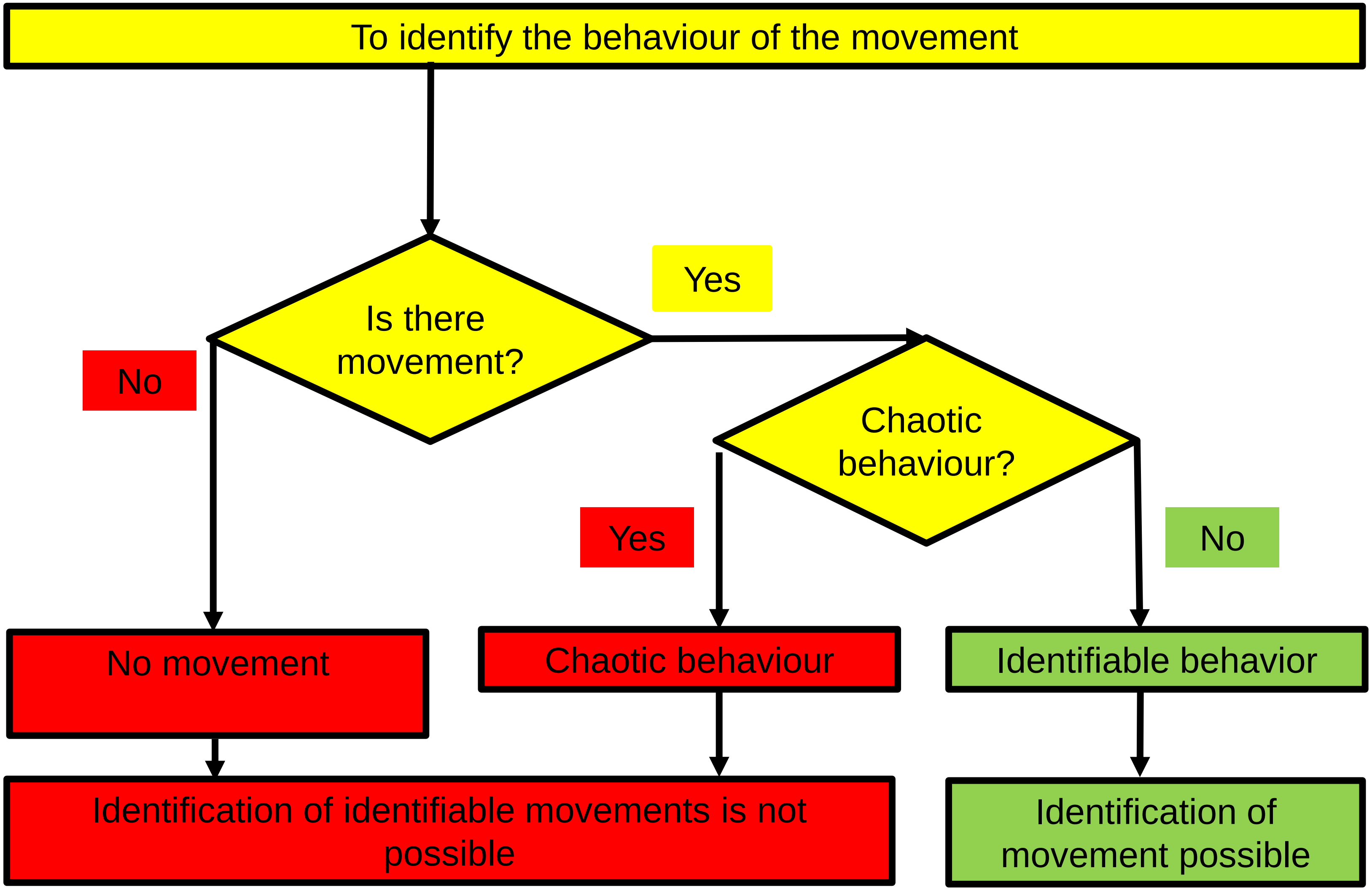
If motion is present without chaotic behaviour, then you can proceed with identification. In selecting the identification process and identification period, it is not yet observed whether there is a stationary or transient response.

These questions are answered through simple mathematics.

Systems can therefore be reliably identified, because in chaotic behaviour and too little movement, the identification of movement can no longer occur. Then alternative measures can specifically be taken such as elimination of identification, error messages etc.

1. Introduction

1.1 Schematic overview



2. Concept of Behavior Indicator of Motion

2.1 Theory

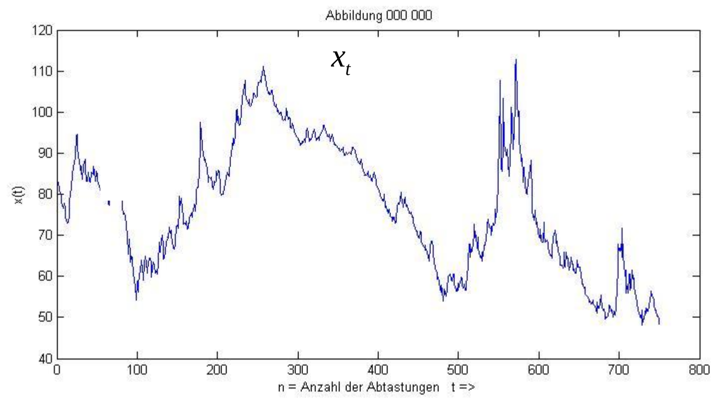
1.Approach: The information of the movement can be found in signals or functions.	$x(t)$ Condition: $\int_0^T \left(\frac{dx(t)}{dt}\right)^2 dt > 0$
2.Scaling: The movements are normalized to the time slice.	$BIM(x(t)) = \frac{\int_0^T \left(\frac{dx(t)}{dt}\right)^2 dt}{\int_0^T (x(t))^2\, dt}$
3. Limit of the maximum movement: Gaussian noise Delta impulse and white noise	$BIM_{max} = \frac{2^2}{\Delta T^2}$ $BIM(x(t)) = \frac{BIM_{max}}{2} = \frac{2}{\Delta T^2}$
1. Level at the crossing: - From the stationary behavior and transient response to chaotic behavior - The balancing behavior begins	$BIM = \frac{BIM_{max}}{2^2} = \frac{1}{\Delta T^2}$ $BIM = \frac{\pi^2}{T^2}$
2.Lower limit When motion is present, BIM is greater than 0	$BIM(x(t)) \rangle\ 0$
3.Definition area	**0 < BIM(x(t)) < 2²/∆T²**

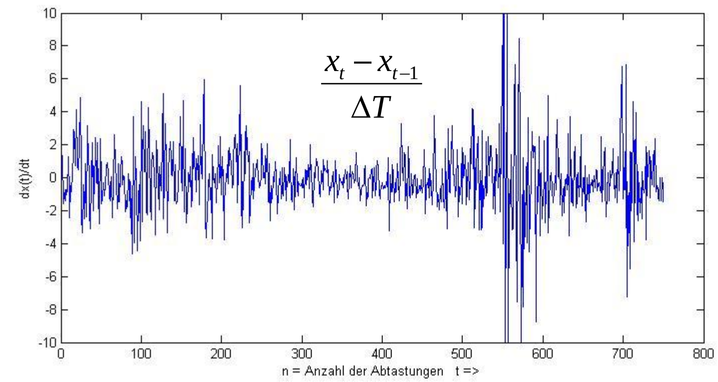
2. Concept of Behavior Indicator of Motion

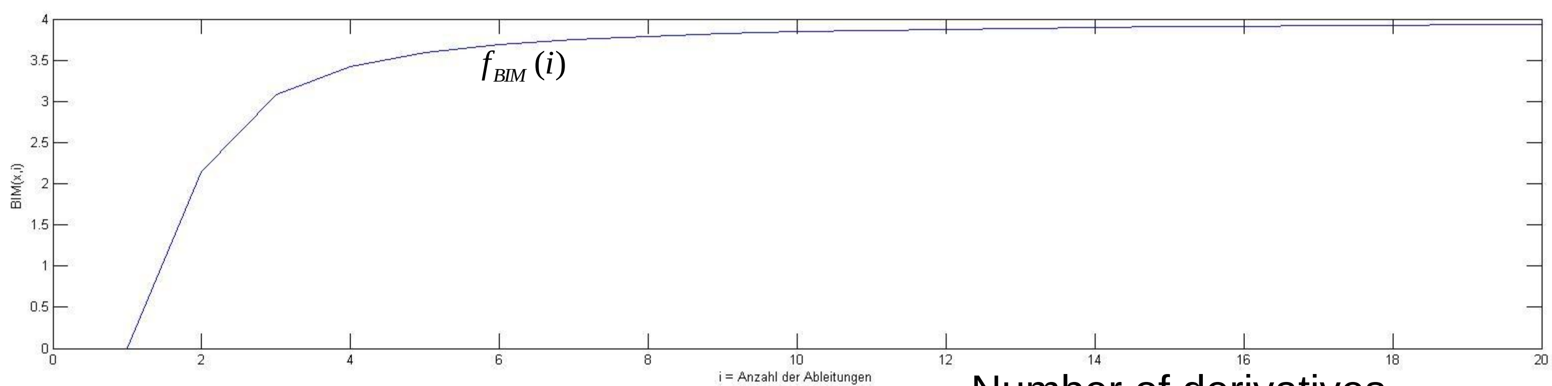
2.2 Heuristic

$$\lim_{\xrightarrow[i \text{ gegen } \infty]{}} f_{BIM\,(x(t))}(i) = \frac{r_{\frac{dx^i}{dt^i}(t,T)\frac{dx^i}{dt^i}(t,T)}(\tau = 0)}{r_{\frac{dx^{i-1}}{dt^{i-1}}(t,T)\frac{dx^{i-1}}{dt^{i-1}}(t,T)}(\tau = 0)} = \frac{2^2}{\Delta T^2}$$

$$BIM(x_i) = \frac{\sum_{2=t}^{T}\left(\frac{x_{i,t} - x_{i,t-1}}{\Delta T}\right)^2}{\sum_{2=t}^{T}\left(x_{i,t}\right)^2} = \begin{cases} \frac{2^2}{\Delta T^2} & \text{max} \\ 0 & \text{min} \end{cases}$$







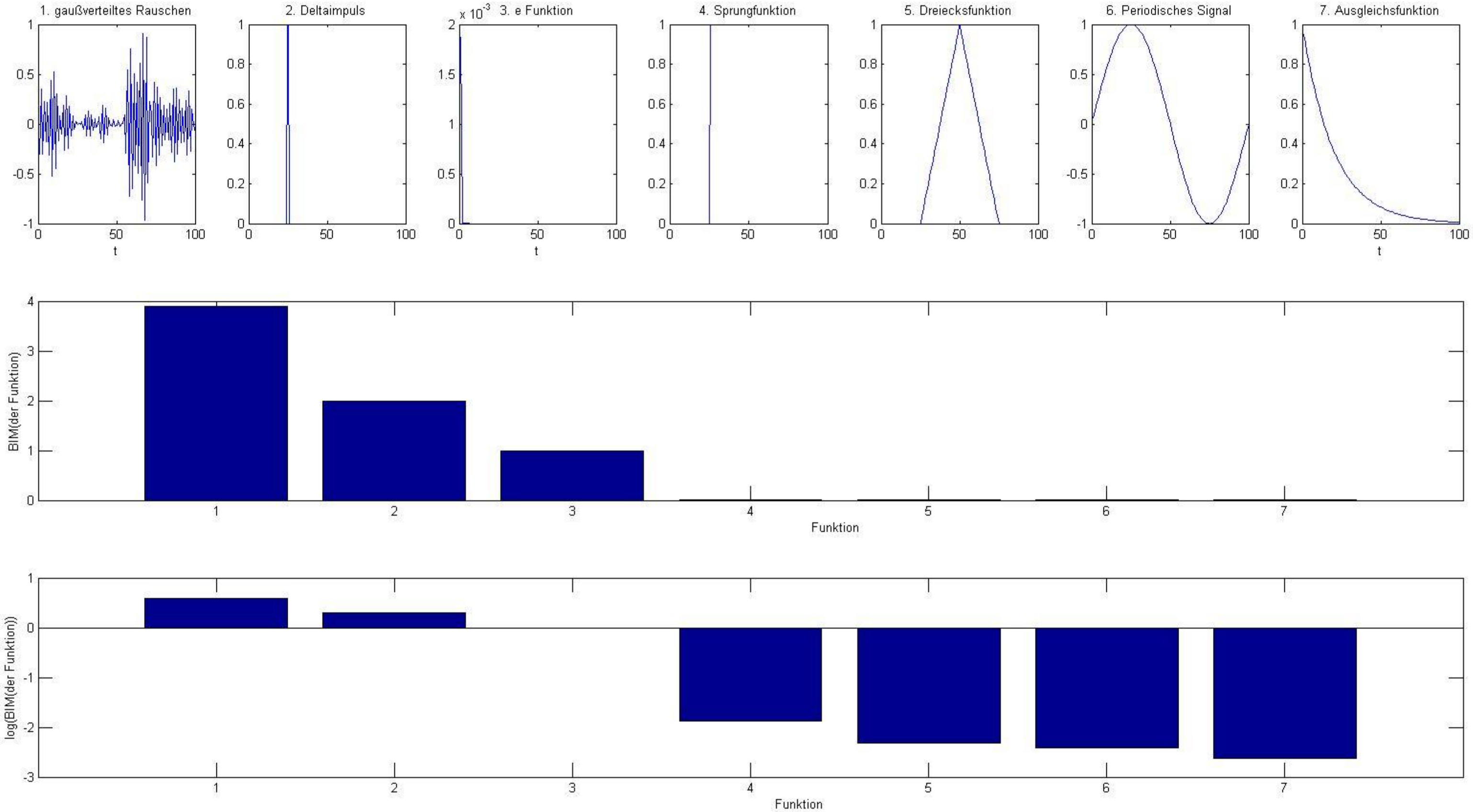
Number of derivatives

2. Concept of Behavior Indicator of Motion

2.3 Heuristic example

$$BIM(x) = \frac{\sum_{2=t}^{T} (x_t - x_{t-1})^2}{\Delta T^2 * \sum_{2=t}^{T} (x_i)^2}$$

Numerical calculation of the signals with noise and the following functions: T = 100 ΔT



3. Proof of Behavior Indicator of Motion

3.1 Domain of movement

$$BIM(x(t,T)) = \frac{\frac{1}{T}\int_0^T \left(\frac{x(t,T) - x(t-\Delta T,T)}{\Delta T}\right)^2 dt}{\frac{1}{T}\int_0^T x(t,T)^2\, dt} = \frac{\frac{1}{\Delta T^2}\int_0^T \left(x(t,T)^2 - 2 * x(t,T) * x(t-\Delta T,T) + x(t-\Delta T,T)^2\right) dt}{\int_0^T x(t,T)^2\, dt} =$$

$$BIM(x(t,T)) = \frac{\int_0^T 2 * x(t-\Delta T,T)^2\, dt - \int_0^T 2 * x(t,T) * x(t-\Delta T,T)\, dt}{\Delta T^2 * \int_0^T x(t,T)^2\, dt} = \frac{2}{\Delta T^2} * \left(1 - \frac{\int_0^T x(t,T) * x(t-\Delta T,T)\, dt}{\int_0^T x(t,T)^2\, dt}\right)$$

The partial expression moves within the boundaries of like a correlation function.

$$-1 < \frac{\int_0^T x(t,T) * x(t-\Delta T,T)\, dt}{\int_0^T x(t,T)^2\, dt} < 1$$

BIM can not be negative. This results in the following domain:

$$0 < BIM(x,T) < \frac{2^2}{\Delta T^2}$$

3. Proof of Behavior Indicator of Motion

3.2 Level of chaotic to stationary behaviour

Determine the upper limit of the step function.

$$x(t,T) = \begin{cases} 0 & für \quad t \langle t_0 \\ 1 & für \quad t_0 \le t \end{cases} \qquad \frac{dx(t,T)}{dt} = \frac{1}{\Delta T}\begin{cases} 0 & für \quad t \langle t_0 \\ 1 & für \quad t = t_0 \\ 0 & für \quad t_0 \langle t \end{cases}$$

Domain $0 \langle t_0 \langle T$

$$BIM(x(t,T)) = \frac{\frac{1}{T}\int_0^T \left(\frac{dx(t,T)}{dt}\right)^2 dt}{\frac{1}{T}\int_0^T x(t,T)^2 dt} = \frac{\int_0^T \left(\frac{\begin{cases} 0 & für \quad t \langle t_0 \\ 1 & für \quad t_0 \le t \end{cases} - \begin{cases} 0 & für \quad t \langle t_0 + \Delta T \\ 1 & für \quad t_0 \le t + \Delta T \end{cases}}{\Delta T} \right)^2 dt}{\int_0^T \left(\begin{cases} 0 & für \quad t \langle t_0 \\ 1 & für \quad t_0 \le t \end{cases} \right)^2 dt} = \frac{\frac{1}{\Delta T^2} * \int_{t_0}^{t_0+\Delta T} (1)^2 dt}{\int_{t_0}^{T} (1)^2 dt}$$

$$BIM(x(t,T)) = \frac{\frac{1}{\Delta T^2} * \int_{t_0}^{t_0+\Delta T} (1)^2 dt}{\int_{t_0}^{T} (1)^2 dt} = \frac{\int_{t_0}^{t_0+\Delta T} 1\, dt}{\Delta T^2 * \int_{t_0}^{T} 1\, dt} = \frac{(t_0 + \Delta T - t_0)}{\Delta T^2 * (T - t_0)} = \frac{\Delta T}{\Delta T^2 * (T - t_0)}$$

$$BIM(x(t,T)) = \begin{cases} \frac{1}{\Delta T^2} & & \max & bei & t_0 = T - \Delta T \\ \frac{1}{T * \Delta T} & bei \quad T >> \Delta T & \min & bei & t_0 = \Delta T \end{cases}$$

3. Proof of Behavior Indicator of Motion

3.3 Level of stationary behaviour to compensatory behaviour

$$x(t,T) = \sin(\omega_T * t) \qquad \frac{dx(t,T)}{dt} = \frac{\cos(\omega_T * t)}{\omega_T}$$

Constraints $\omega_T = \frac{2\pi}{T}$

$$BIM(x(t,T)) = \frac{\frac{1}{T}\int_0^T \left(\frac{dx(t)}{dt}\right)^2 dt}{\frac{1}{T}\int_0^T x(t,T)^2\, dt} = \frac{\int_0^T \left(\frac{d\sin(\omega_T * t)}{dt}\right)^2 dt}{\int_0^T \sin(\omega_T * t)^2\, dt} = \frac{\int_0^T (\omega_T * \cos(\omega_T * t))^2\, dt}{\int_0^T \sin(\omega_T * t)^2\, dt}$$

$$BIM(x(t,T)) = \frac{\omega_T^2 * \left[\frac{\omega_T}{2} * (\omega_T * t + \sin(\omega_T * t) * \cos(\omega_T * t))\right]_0^T}{\left[\frac{\omega_T}{2} * (\omega_T * t - \sin(\omega_T * t) * \cos(\omega_T * t))\right]_0^T} = \frac{\omega_T^2 * [\omega_T * t + \sin(\omega_T * t) * \cos(\omega_T * t)]_0^T}{[\omega_T * t - \sin(\omega_T * t) * \cos(\omega_T * t)]_0^T}$$

$$BIM(x(t,T)) = \frac{\omega_0^T * ([\omega_T * T + \sin(\omega_T * T) * \cos(\omega_T * T)] - [\omega_T * 0 + \sin(\omega_T * 0) * \cos(\omega_T * 0)])}{[\omega_T * T - \sin(\omega_T * T) * \cos(\omega_T * T)] - [\omega_T * 0 - \sin(\omega_T * 0) * \cos(\omega_T * 0)]}$$

$$BIM(x(t,T)) = \frac{\omega_T^2 * [\omega_T * T + \sin(\omega_T * T) * \cos(\omega_T * T)]}{[\omega_T * T - \sin(\omega_T * T) * \cos(\omega_T * T)]} = \frac{\omega_T^2 * \left(\frac{2\pi}{T} * T + \sin\left(\frac{2\pi}{T} * T\right) * \cos\left(\frac{2\pi}{T} * T\right)\right)}{\frac{2\pi}{T} * T - \sin\left(\frac{2\pi}{T} * T\right) * \cos\left(\frac{2\pi}{T} * T\right)} = \frac{\omega_T^2 * \frac{2\pi}{T} * T}{\frac{2\pi}{T} * T} = \omega_T^2 = \frac{2^2 \pi^2}{T^2}$$

$$BIM(x(t,T)) = \frac{2^2 \pi^2}{T^2}$$

3. Proof of Behavior Indicator of Motion

3.4 e-Function example

$$x(t,T) = e^{t*\omega_T} \qquad \frac{dx(t,T)}{dt} = \omega_T * e^{t*\omega_T}$$

$$BIM\,(x(t,T)) = \frac{\int_0^T \left(\frac{dx(t)}{dt}\right)^2 dt}{\int_0^T \left(x(t)\right)^2 dt} = \frac{\int_0^T \left(\frac{de^{t*\omega_T}}{dt}\right)^2 dt}{\int_0^T \left(e^{t*\omega_T}\right)^2 dt} = \frac{\omega_T^2 * \int_0^T \left(e^{t*\omega_T}\right)^2 dt}{\int_0^T \left(e^{t*\omega_T}\right)^2 dt} = \omega_T^2 = \frac{2^2 * \pi^2}{T^2}$$

$$BIM\,(x(t,T)) = \frac{2^2 * \pi^2}{T^2}$$

3. Proof of Behavior Indicator of Motion

3.5 Delta impulse example

$$x(t,T)=\begin{cases}0 & \text{für} \quad t \langle t_0 \\ 1 & \text{für} \quad t = t_0 \\ 0 & \text{für} \quad t_0 \langle t\end{cases}$$

$$\frac{dx(t,T)}{dt}=\frac{1}{\Delta T}\begin{cases}+0 & \text{für} \quad t \langle t_0 \\ +1 & \text{für} \quad t = t_0 \\ -1 & \text{für} \quad t = t_0+\Delta T \\ +0 & \text{für} \quad t_0+\Delta T \langle t\end{cases}$$

Domain $0 \langle t_0 \langle T$

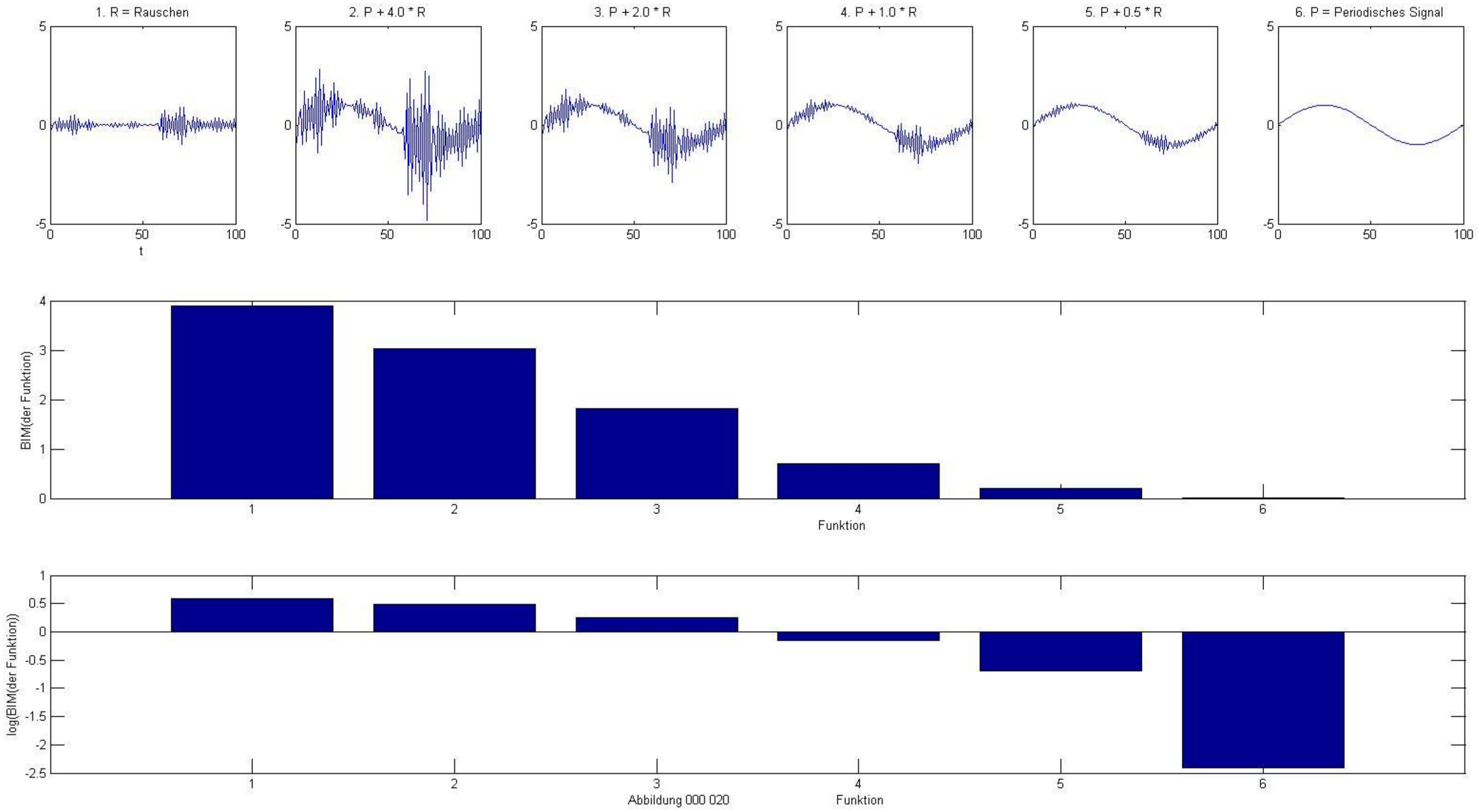
$$BIM(x(t,T))=\frac{\frac{1}{T}\int_0^T\left(\frac{dx(t,T)}{dt}\right)^2 dt}{\frac{1}{T}\int_0^T x(t,T)^2 dt}=\frac{\int_0^T\left(\frac{1}{\Delta T}\begin{cases}+0 & \text{für} \quad t \langle t_0 \\ +1 & \text{für} \quad t = t_0 \\ -1 & \text{für} \quad t = t_0+\Delta T \\ +0 & \text{für} \quad t_0+\Delta T \langle t\end{cases}\right)^2 dt}{\int_0^T\left(\begin{cases}0 & \text{für} \quad t \langle t_0 \\ 1 & \text{für} \quad t_0 \le t\end{cases}-\begin{cases}0 & \text{für} \quad t \langle t_0+\Delta T \\ 1 & \text{für} \quad t_0 \le t+\Delta T\end{cases}\right)^2 dt}=\frac{\frac{1}{\Delta T^2}*\left(\int_0^{t_0}(1)^2 dt+\int_{t_0+\Delta T}^{T}(1)^2 dt\right)}{\int_{t_0}^{t_0+\Delta T}(1)^2 d}$$

$$BIM(x(t,T))=\frac{\frac{1}{\Delta T^2}*2}{1^2}=\frac{2}{\Delta T^2}$$

4. Effectiveness of Behavior Indicator of Motion

4.1 Heuristic example of chaotic behavior to stationary behavior

The signal behaviour of noise in the lift figure is shown for stationary signals of different types, for example, a sine function shown in the right figure.



4. Effectiveness of Behavior Indicator of Motion

4.2 Heuristic examples of calculations

Functions:	Period under consideration $T = 100\ \Delta T >> \Delta T$ BIM(x(t, T))	Behaviour
1.) Gaussian noise	$4.00000 / \Delta T^2 = BIM_{max} = 2^2 / \Delta T^2$	chaotic
2.) Delta Impulse	$2.00000 / \Delta T^2 = BIM_{max} / 2$	chaotic
3.) Distributed noise	$2.00000 / \Delta T^2 = BIM_{max} / 2$	chaotic
4.) e-Function	$1{,}00000 / \Delta T^2 = BIM_{max} / 4$	observable
5.) Step function		
- Maximum value:	$1{,}00000 / \Delta T^2 = BIM_{max} / 4$ bis	observable
-:Minimum value	$0{,}04000 / \Delta T^2 < BIM_{max} / 4$	observable
6.) Triangular function as an example	$0{,}12000 / \Delta T^2 < BIM_{max} / 4$	observable
7.) Rectangular function as an example	$0{,}08000 / \Delta T^2 < BIM_{max} / 4$	observable
8.) Sine function	$0{,}00394 / \Delta T^2 < BIM_{max} / 4$	observable

4. Effectiveness of Behavior Indicator of Motion

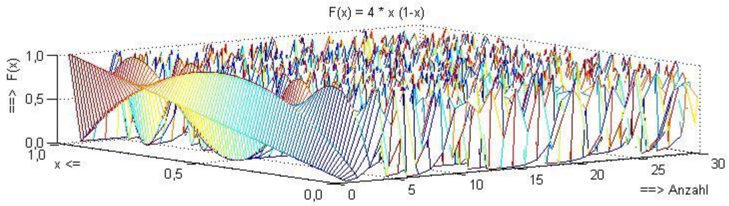
4.3 Example with a fig tree function

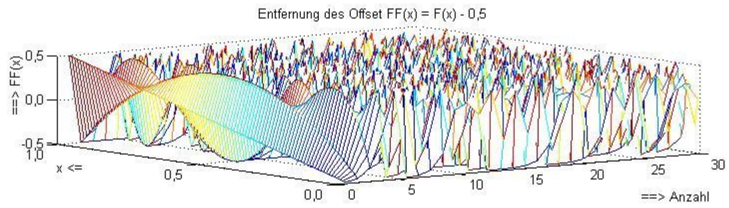
$F(x) = 4 * x * (1 - x)$ $\qquad$ $Domain: 0 \le x \le 1$

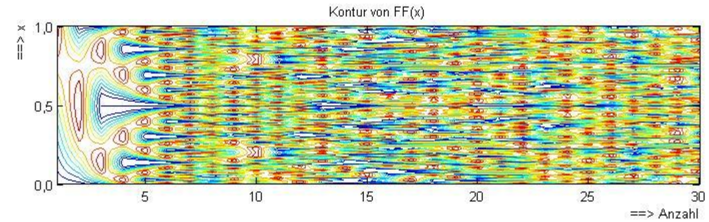
Begin with x constant from 0 to 1
e.g. with 1 / 100th of quantization are the cuts
$e.g.\ Number = 5;\ F(x)_{Number=5} = F_{Number=4}(F_{Number=3}(F_{Number=2}(F_{Number=1}(4*x*(1-x$

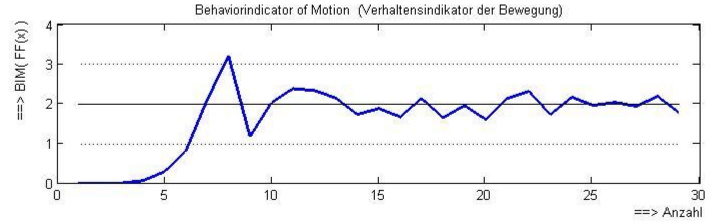
$$BIM(F(x)) = \frac{\int_0^T \left(\frac{F(x)}{dx}\right)^2 dx}{\int_0^T (F(x))^2\ dx}$$

$$BIM_{Number}(F_{Number}(x)) = \frac{\int_0^T \left(\frac{F_{Number}(x)}{dx}\right)^2 dx}{\int_0^T (F_{Number}(x))^2\ dx}$$





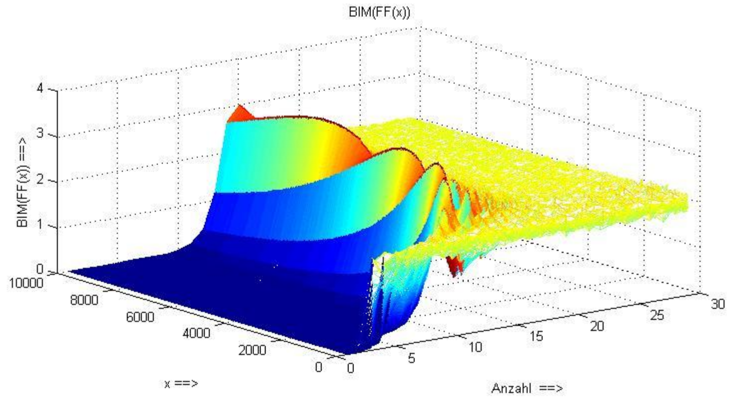


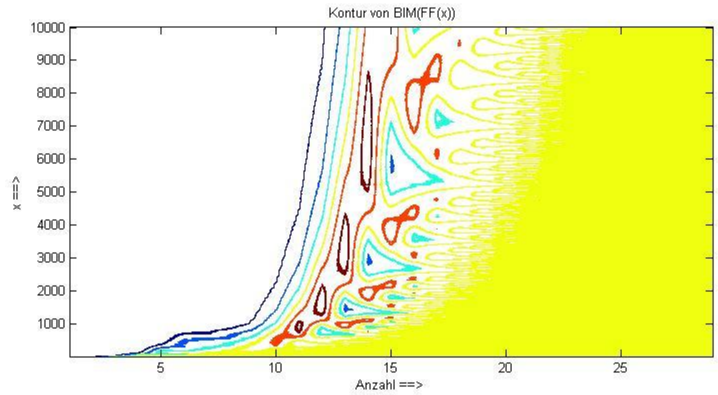


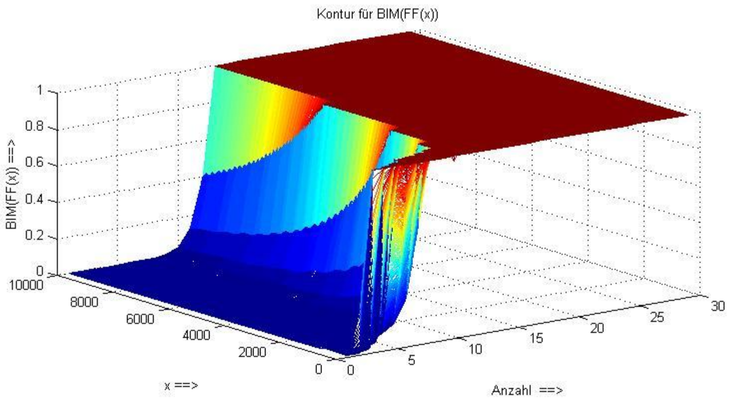
4. Effectiveness of Behavior Indicator of Motion

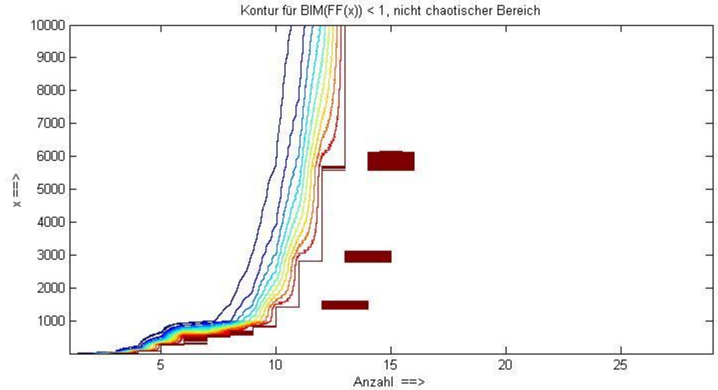
4.3 Example with a fig tree function

Begin with x constant from 0 to 1 with 1 / (11 to 10.000) Quantizations are the sections





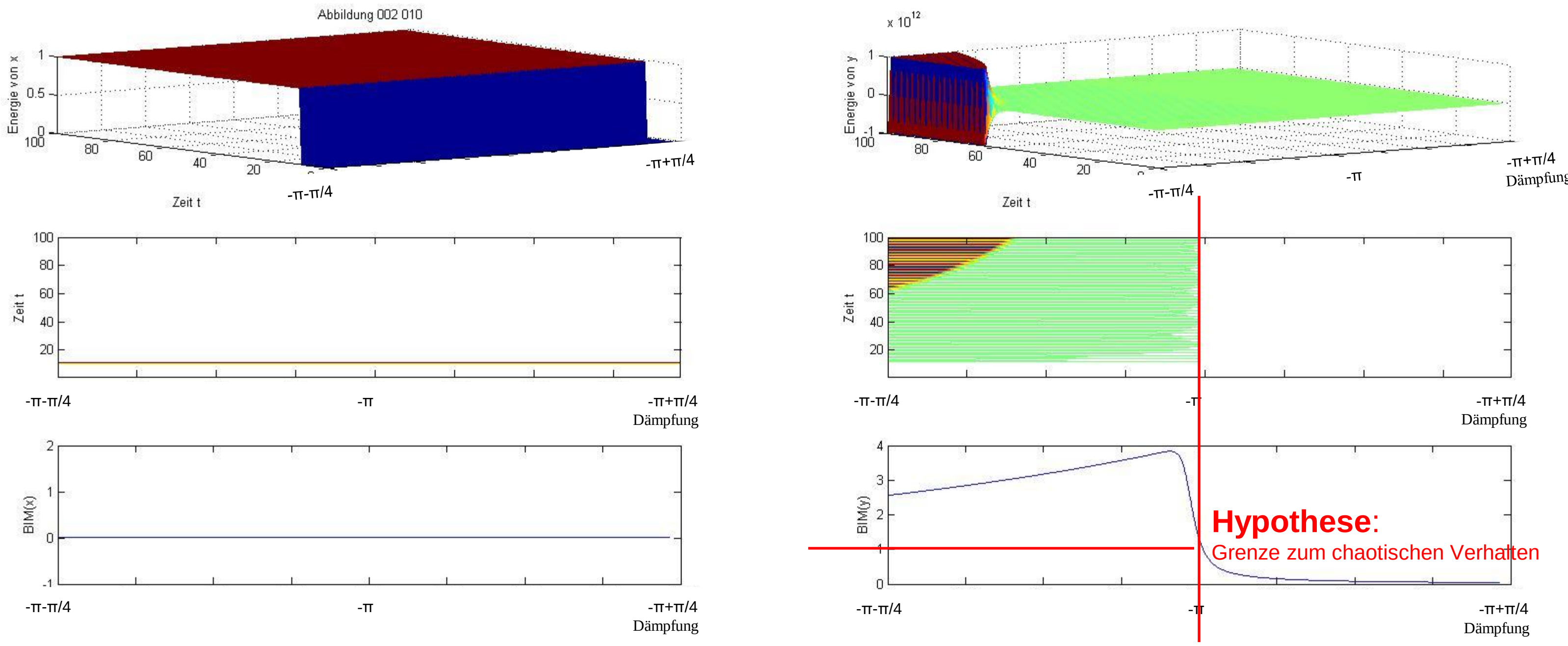




4 Effectiveness of Behaviour Indicator of Motion

4.4 Example of a System 1. Order with the damping -π ±π/4

- The transition to chaotic behaviour is with damping of -π.



Freqenzbereich: $G(s) = \frac{K}{1+s\,T}$

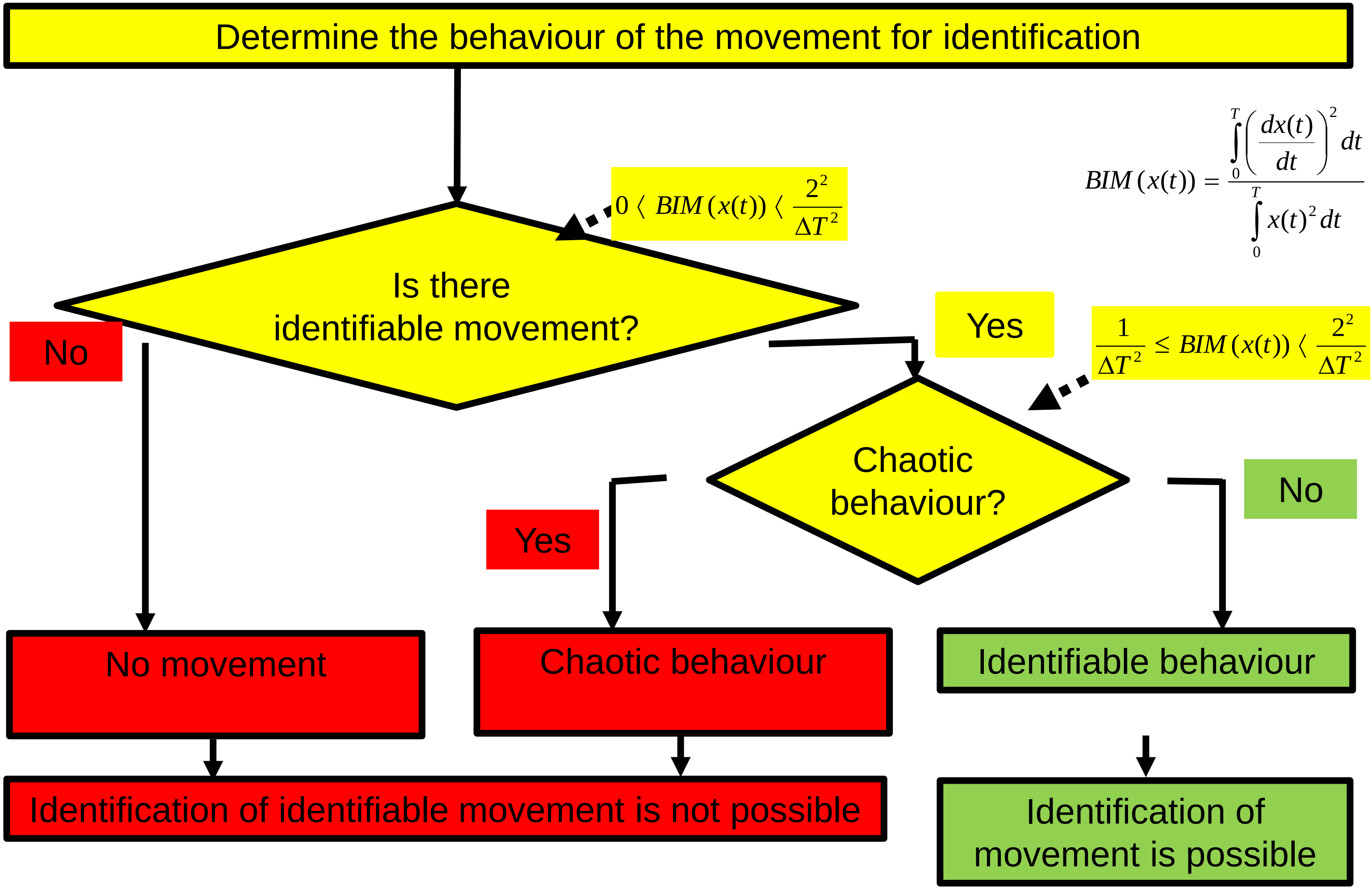
Sprungantwort x(t): $y(t) = K\left(1 - e^{-\frac{t}{T}}\right) x(t)$

Eingangssignale: $x(t) = \begin{cases} 0 & \text{für } t < 10\Delta T \\ 1 & \text{für } t \geq 10\Delta T \end{cases}$

Integrationszeitraum: $T_I = 100\ \Delta T$

4. Effectiveness of Behavior Indicator of Motion

4.5 Schematic overview



4.5 Schematic overview

Identification of identifiable movement is not possible

↓

Measures:

- Signaling fault (red lamp)
- Alarm
- Bring movement into a safe state
- Save data for analysis of the fault
-

Identification of identifiable movement is possible

↓

Normal operation:

- Signaling operation (green lamp)
- The signals are observable
- Movement is in a safe operating condition
- Routinely monitor movement
- Save data for system optimization as required

5. Summary

With this work, the identification of movements is improved by the following procedure:

System identification and stability monitoring:

Hypothesis:
Shifting of the classical stability limit at the transition to chaotic behaviour

Testing with the Behavior Indicator of Motion (BIM), how and if the signals are identified at all.

Outlook:

Investigation of the limits of chaotic behavior with the Behavior Indicator of Motion.

Determine the period from which the stationary signal can be identified with BIM_T.

A fast adaptive controller can be built that can adjust to the chaotic behaviour border on the stability boundry.

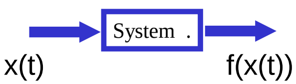
Behaviour Indicator of Motion

End

Thank you for your attention

5. Summary

5.1 Definition classifications

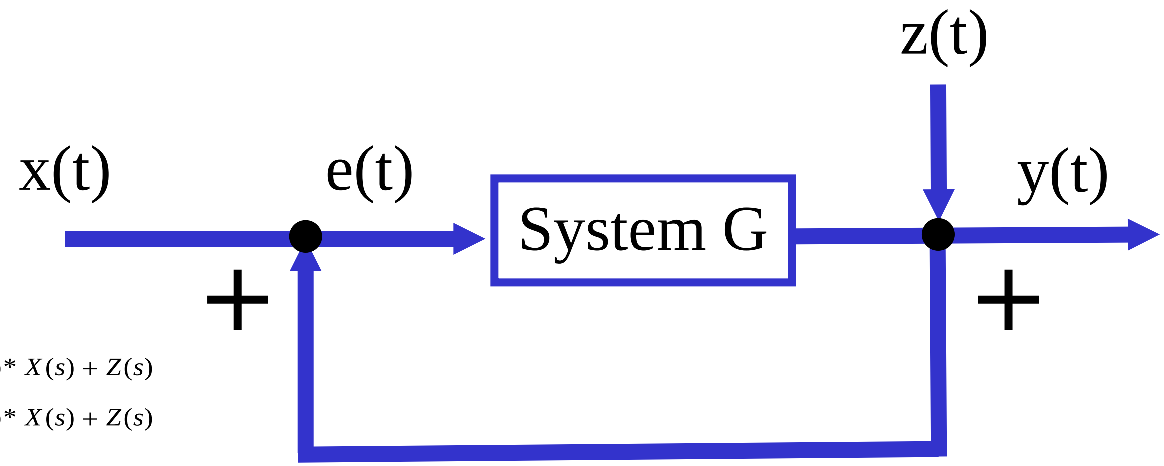
Scientist	Aleksandr M. Lyapunov	Reiner Kühn
Term	General Problem of Stabilitly of Motion	Behaviour Indicator of Motion
Defined in	1890	2012
Formula: 	$\mid f(x(t)) \mid \leq K$	$\frac{2^2\ \pi^2}{T^2} \langle \frac{\int \left(\frac{df(x(t))}{dt} \right)^2 dt}{\int f(x(t))^2 dt} \langle \frac{1}{\Delta T^2}$
Dimension	Dimensions involved	Time dependant
Domain	New values determined in each case	Fixed limit
Time behaviour	Dependant on timepoint	Universal
Quality	Robust estimation of signals	Differentiation from a threshold
Calculation	Value formation nonlinear reflection	Auto-correlation functions, which are included in the adaptive system identification calculation

5. Summary

5.1 Definition classifications

Scientist	**Nowton**	**Nowton**	**Nowton**	**Nowton**	**Aleksandr M. Lyapunov**	**Reiner Kühn**	**Reiner Kühn**
Term	**Law of Motion** **1**	**Law of Motion** **tow**	**Law of Motion** **3**	**Law of Motion** **4**	General Problem of Stabilitly of Motion	Behaviour Indicator of Motion	Satability Law of Motion
Defined in					ca. 1890	ca. 1993 (Invers.)	ca. 2014
Formula: x(t) f(x(t))							
Dimension					Dimensions involved	Time dependant	Time dependant

BIM Analysis with an error z(t):



$Y(s)=G(s)*X(s)+Z(s)$

$Y(s)=G(s)*X(s)+Z(s)$

4.7 Time indicator, appears from the steady-state behavior for the first time

Mathematical Defintion

If you increase T, step by step, ID is also steadily greater to BIM = 1 and stationary behaviour is possible, then we have a Value Time Indicator ($TBIM_T$) if BIM = 1.

$$BIM(x(t,T_{ID})) = \frac{\frac{1}{T_{ID}}\int_0^T \left(\frac{x(t,T_{ID}) - x(t-\Delta T,T_{ID})}{\Delta T}\right)^2 dt}{\frac{1}{T_{ID}}\int_0^T x(t,T_{ID})^2 dt} = \frac{1}{\Delta T^2}$$

From T = 0 to BIM_T(BIM(x(t,T)=1) stationary behaviour can not occur, rather transient effects.

Properties:

T < T_{ID}(BIM(x(t,T)=1)	no stationary behaviour
T ≥ T_{ID}(BIM(x(t,T)=1)	stationary behaviour can occur
T >> T_{ID}(BIM(x(t,T)=1) * robustness factor	stationary behavior occurs with high probability

4.7 Why is BIM from 0 to 1 / 4 * ΔT² not a chaotic signal?

Mathematical definition

$$BIM(x) \quad \rangle \quad BIM(y)$$

Ensure the BIM(y) < 1 / ΔT²

4History

$$BIM(x) \quad \rangle \quad BIM(y)$$

Ensure the BIM(y) < 1 / ΔT²